DOCUMENT RESUME

ED 246 924 JC 840 303

AUTHOR Maurer, Stephen B.
TITLE The Lessons of Williamstown.
PUB DATE Jul 84
NOTE 12p.; Paper presented at the Sloan Foundation Conference on New Directions in Two-Year College Mathematics (Atherton, CA, July 11-14, 1984).
PUB TYPE Speeches/Conference Papers (150) -- Viewpoints (120)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Calculus; *College Mathematics; *Curriculum Development; Educational Needs; Higher Education; *Mathematics Curriculum; Two Year Colleges; *Undergraduate Study
IDENTIFIERS *Discrete Mathematics

ABSTRACT

In July 1982, a conference was held at Williams College in Williamstown, Massachusetts, to discuss the need for more discrete mathematics in the first two years of the core undergraduate mathematics curriculum. This paper reviews the recommendations that were made during the conference, the progress that has been made in the implementation of those recommendations, and the resistance and doubt that met the proposed changes. After briefly reviewing the areas of agreement and disagreement that emerged during the conference, the paper discusses the current status of undergraduate mathematics education in the United States, pointing to increasing interest in discrete mathematics, the specific activities of colleges funded to develop new undergraduate programs, and the revised curriculum at Dartmouth College. Next, reasons for the apparent resistance to curricular change are discussed, with respect to proposals for emphasizing discrete mathematics and reducing the calculus core to 1 year. This discussion is followed by a number of personal observations about what college freshmen and sophomores do and do not need to know about mathematics, and about the implications of these educational needs for curriculum development. Finally, the relevance of the debate over discrete mathematics for two-year colleges and the options available in developing math courses for non-transfer oriented students are explored. (HB)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED246924

THE LESSONS OF WILLIAMSTOWN

Stephen B. Maurer
Associate Professor
Swarthmore College
Swarthmore, Pennsylvania

Presented at the Sloan Foundation Conference on
New Directions in Two-Year College Mathematics
Atherton, California, July 11-14, 1984

JC 840 303

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
S. B. Maurer

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
☐ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

THE LESSONS OF WILLIAMSTOWN

Stepnen B Maurer
Swarthmore College & the Alfred P Sloan Foundation

The title was suggested by Don Albers. It refers to the conference held in July, 1982, at Williams College to discuss the need for more discrete mathematics in the first two years of the core undergraduate mathematics curriculum. This conference is a follow-up to that one, in the view of both the organizers and the Sloan Foundation, which has funded both. Consequently, it is appropriate to review the lessons of Williamstown as we begin in Menlo Park.

Unfortunately, the title has caused me a lot of agony. I'm not sure I have any lessons to tell! This should not be surprising. Given that a major overhaul of undergraduate mathematics was proposed, two years is a short time to see much happen. Perhaps I should have changed my title to "Status Report". Even here I'm not sure I have much useful to say. In this business, even two years is a short time to see any trends. But the fact is, part of the reason there hasn't been much change is that there is a lot of resistance — not so much outright objection as severe doubt that the proposed changes can be effected successfully.

In any event, here's what I'm going to do. First I will say briefly what happened at Williamstown and where things stand now. Then, in a long middle section, I will give my analysis and reaction to the doubts. (Perhaps I should have titled this paper "Everything I am Thinking about the Discrete Math Project Now".) Finally, I will close with some brief thoughts about how all this pertains to two-year colleges.

Williamstown

The desire for change at Williamstown was unanimous. But then, this shouldn't be surprising. Had the title of the conference been "The Continuing Importance of Calculus", there still would have been a consensus for change — nobody is ever satisfied with the calculus course!

More significant was the agreement that there <u>should</u> be more discrete math in the first two years of the college curriculum. But there was less agreement as to <u>what</u> discrete math to add and how. Should it be an emphasis on formal algorithms and recursion? Should it be math modeling? Are new courses necessary or is it enough to replace a few weeks in the current calculus and linear algebra sequence? All these were suggested.

There was also disagreement on what could be taken out. Could calculus be reduced to one year or not? Not surprisingly, the conferees from different fields (physics, engineering, computer science, social science) held different views. (Here and elsewhere, when I refer to a one-year calculus, I mean calculus including multivariate. That is, I refer to the material that usually forms a three-semester sequence or, in some places still, a four-semester sequence.)

Current Status

Interest in the mathematical community in this issue has grown by leaps and bounds. For instance, at MAA meetings talks and panels on discrete math in the first two years get SRO audiences. I sense that there is finally a pretty good understanding in the community of how the proposed algorithmic discrete math course differs from discrete structures and finite math. There is also a growing awareness that the issue is much broader than service to the CS community.

There has also been some specific activity. Based on Williamstown, the Sloan Foundation invited several schools to make proposals for developing a new underclass curriculum, and six schools were funded at $40,000 each: Colby College, University of Delaware, University of Denver, Florida State University, Montclair State College, and St Olaf College. Also, the MAA set up a Discrete Math Panel, headed by Prof Martha Siegel of Towson State (MD), to write a report much in the style of the Mathematical Sciences Report of a few years ago (part descriptive, part prescriptive). A preliminary report should be out this fall, a final report a year later.

But other than this, I think I have to say that not much is happening. Except for the Sloan schools, I haven't heard much about institutions revamping their curricula. Even the Sloan schools, which are in the middle of two and three year schedules, are having some trouble. They have no problem introducing a discrete course (just lots of work to produce text materials), but their commitment to reducing the calculus seems to be eroding. Even at the school which has placed the highest priority on how they redo the calculus (Colby, where the first year of the project was devoted solely to this, and a discrete course will be introduced next year for the first time) there is now the feeling that their one-year calculus course is really successful only for students who have had some calculus in high school.

I would like to point out an important exception to my statement that not much is happening elsewhere. Dartmouth has voted to change its program, effective for September's freshmen, to one year of calculus and one year of discrete math and linear algebra. In more detail, the core math program in the first two years for scientifically inclined students will be: a term of calculus followed by a term of discrete math followed by two terms which interweave calculus, including multivariate, and linear algebra. This sequence will be required for entrance to the math department. The first two terms of the sequence will be prerequisites for the first computer science course. (This first course is quite sophisticated and assumes lots of programming experience. There are no programming courses per se at Dartmouth, but many courses, including the first calculus term and the discrete term of the new math sequence, include lots of computing.) In order to make this change politically acceptible to all departments, engineering and some science students will be allowed to take the second term of calculus without the discrete term, and there will be a special introduction to computer science for those with this background. However, the computer science department hopes this special course will be temporary (until engineers see the usefulness of discrete math). In any event, no student will be able to go any further in the mathematics or computer science curricula without that second term discrete math course.

Also, it turns out that at least two schools -- Grinnell and Williams --

are already doing calculus in one year. Indeed, they've been doing this for several years, quite aside from the discrete math issue. Both these institutions are now instituting semester discrete math courses to be taken after calculus, or for some students, after a semester of calculus.

So calculus certainly can be reduced, making room for discrete math, at least at certain sorts of schools. We have several constructive proofs!

Back to Dartmouth: there are actually two things going on. There is the new math curriculum for math and science majors, discussed above, and there is a new version of the finite math service course. Rather than writing a 4th edition of Kemeny Snell and Thompson [1], Kemeny Kurtz and Snell have written companion notes [2] introducing computer programming and providing lots of nontrivial mathematical examples to be carried out on the computer. (Write to Kemeny for a copy.) In other words, the course is algorithmic in the sense that students write and use algorithms, but they don't confront the mathematics of algorithms, except informally. Although this is not the curricular change the Williamstown conference had in mind, it may well be a change which will succeed. (In order to make room for the new material, the course is also extended to two terms.)

Ruminations

Let me begin by returning to Williamstown for a moment. I may have given the false impression that it didn't accomplish very much. As you can imagine, during my two years at Sloan I have been invited to quite a few conferences on what to do about this or that. In retrospect, Williamstown was one of the very best. Most of the discussion was on specifics; there was little vague talk. Moreover, specific proposals came out. Three curricula were proposed in the papers written before the conference, and two more were developed as consensus documents during the meeting. (One was for a unified discrete/continuous two-year sequence, the other for a curriculum of separate courses.) If you read these consensus documents, you will perceive much less disagreement than I intimate above. However, as I see it now, the reason agreement on what to put in was easy is that it was agreed to put almost every discrete suggestion in. The way it was agreed to take a lot of calculus out was to make the statement, almost offhand, that those who needed the things omitted could learn them later. This form of agreement may have been too facile.

Nonetheless, I do feel the contributions at Williams were high quality. To prepare myself to write this paper (or maybe it was to postpone writing it) I reread the Proceedings [3]. Yet again they provided me with new insights and fresh ideas. For instance, I was struck this time by how much the proposals involved integration of discrete and continuous ideas, e.g., teaching difference and differential equations concurrently. Yet, only one of the course development proposals solicited by Sloan proposed to try an integrated curriculum, and it was not well enough thought out to be funded. It seems that an integrated approach is the natural way one thinks to do things in the best of possible worlds, but that this approach is too great a leap when we actually commit ourselves to action.

As long as I am talking about new insights from rereading, let me mention another case. I recently had occasion to look carefully again at that classic

hard calculus book of the 60's: Apostol [4]. My new insight was: it's not a calculus book. It has all sorts of things in it that we discrete math advocates have been talking about — combinatorics, induction, probability and statistics, difference equations, numerical analysis. Granted, much of this is in the second volume, certain current perspectives and topics are completely lacking (the algorithmic perspective, the topic of graph theory), and it is all wrapped around calculus as the core. Nonetheless it's clear that Apostol wasn't trying to teach just calculus; he was trying to teach mathematics, as an integrated two-year course, according to the best judgment he could make at the time. This was very much the spirit of Williamstown.

Let me turn, finally, to what I perceive as the reasons for resistance to change.

Discrete Mathematics. The main problem here remains the lack of a book. Faculty like the idea that there might be a core of discrete math of value to almost everyone who will use math, and which is teachable at the freshman/sophomore level, but they'll believe it when they see it. The second problem is that, except for computer science, no serviced discipline is asking for discrete math. The people who are using discrete math for research, say, in management science or sociology are not yet asking that this be taught to their incoming students. All this is mirrored by the fact that there are plenty of books for freshmen and sophomores with titles like "Discrete Math for Computer Science", but none with "for Computer Science" deleted.

Actually, I did finally see the shorter title a few months ago, by Richard Johnsonbaugh of DePaul [5]. Johnsonbaugh makes the point in his preface that discrete math is good for a lot of people, though his topics still have a fairly exclusive CS bent.

In any event, a lot more books without CS in the title are being written, by people at the Sloan funded schools, at Dartmouth, and elsewhere. If Tony Ralston and I would only stop going around giving talks, we might even finish our own book.

Calculus in One Year. This hasn't happened because most mathematicians simply don't believe it can be done. And frankly, those of us advocating it have been a bit naive — at least I have. When people have asked me how it is to be done, I have made statements like, "You really could cut back on methods of integration or related rates". They counter by pointing out that they have already cut back substantially on these topics, and at most they would gain two or 3 days by cutting further. Furthermore, they point out that colleagues in the traditionally serviced departments are already unhappy that certain material their students used to know from calculus can no longer be assumed. When I say, "Couldn't you cover curve sketching by computer graphics rather than calculus?", they counter by arguing that it will take just as long for students to master the software and develop "function sense" this way as by the traditional method. When I say that symbolic computation software will drastically reduce the amount of time needed for drill, they are doubtful. Also, they don't have the software.

[A status report on such software: The fact that it exists does seem to have become well known. A "Computer Algebra" conference was held in April in New York at which some distinguished mathematicians lectured on how they have

started using computer algebra in their research. The conference was expected to draw a few people: 400 came. As for the software itself, the number of powerful systems is growing — MACSYMA, SMP, Maple, Cayley, ScratchPad, Reduce -- and the cost to academic institutions is down to a few K or less. (The basic cost of MACSYMA for a Vax is now $500.) But it's still true that these are big systems. They generally need a mainframe, on which they take a lot of space and/or run slowly if supporting several users. The only serious system for micros, MuMath, is much less powerful, and not very user friendly. The hand-held machine foreseen by Wilf [6] is nowhere in sight. In short, symbolic computation is not yet convenient for colleges. Also, textbooks geared to teaching with it simply don't exist.]

At any rate, the doubts expressed about doing calculus in one year have some merit. Such doubts have pretty well convinced me that 1) calculus cannot be shortened by nibbling away here and there, and 2) calculus is not likely to be substantially condensed by a productivity gain due to technology.

I still do think calculus can be reduced to one year in the first 2, but one has to take a more radical approach. I will speak to this below, but first I want to deal with a different sort of objection.

The further objection is philosophical. I can best explain it by an example. In the last year, I have twice heard a distinguished analytic number theorist criticize Donald Knuth as follows. In "The Art of Computer Programming, Vol 1" [7], page 58, Knuth gives 6 identities for sums of products of binomial coefficients. He relegates the proofs to the exercises; his hints suggest that he regards the proofs as varying from identity to identity, and being a bit tedious. The number theorist says that, if Knuth only knew the theory of hypergeometric series better, he would recognize that all 6 formulas fall out at once. The general point, of course, is that there is a long tradition in analysis, and one who doesn't know it all may have to reinvent the wheel, or may never discover that there is a wheel. If calculus education is diminished, all scientific progress will thus be slowed.

A related argument goes as follows. Haven't we advocates of discrete math already gotten what we need? Isn't discrete math already a popular offering at the upperclass level? The next generation of leading mathematicians are already being exposed to it.

I feel these arguments involve an important confusion. I think we all agree:

It would be best if everyone knew everything!

Unfortunately, that will never happen. Well then, can't we at least insist that future mathematicians should know a lot of analysis? By all means, we can, do, and should. But, at some point, there are diminishing returns. It's not clear that it would have been worth it for Knuth (who knows a great deal of analysis) to take an extra course in hypergeomtric functions if all it saved him and his readers were a few pages of calculations.

As to the upper level discrete offerings, if all we were concerned about were the best students going into math, or even if we were only concerned about math majors, then it would be enough to have discrete math at the upper level.

(It would also be enough to have continuous math at the upper level.) But there is a whole population to be educated; faculty at two-year colleges know this better than most. Moreover, students have the annoying habit of dropping out of mathematics at the most inopportune times -- sometimes after only one college course. Some never sign up at all.

In short, we educators are confronted with the following optimization problem. If 90% of students need to know something about A, 80% need to know something about B, 70% need to know about C, etc., and furthermore, most students take a limited amount of mathematics, in what order should topics be taught to maximize the amount of needed knowledge which students have? It seems pretty clear that a greedy algorithm is called for: to the extent that logicial sequencing permits, topics should be taught in the order of their breadth of usefulness. (Caution: I don't have a theorem here, because I haven't fully specified the model. For instance, it makes a difference which students drop out when.)

Let me return now to the matter of calculus in one year. There is a whole tradition of expectations about what students will learn in their first two years of mathematics. For instance, it is assumed they will learn integration by parts, but not that they will learn any probability. Consequently, a teacher in a later math course, or in another discipline, will do an integration by parts without starting from scratch, but must start from scratch with any probability. Similarly, a physics professor can assume that a student with 3 semesters of calculus knows Green's Theorem, but an economics professor cannot assume the student has even heard of Euler's Theorem on homogeneous functions.

If a major curriculum change is to take place, in particular, if calculus is to become one year, then there must be a conscious decision to make a wholesale change in such expectations. A lot of material must either be kicked up (it becomes grist for junior/senior math courses) or kicked out (it becomes an application which other departments must teach as needed). After some initial objections from colleagues who lose favored status, this all ought to be workable so long as everybody knows the score.

Here are my ideas about things to kick up or out. I've searched my soul, and without shame I can say to my colleagues, in mathematics and other departments, that:

1) Freshmen and Sophomores need only know about the simplest integration methods (simple substitutitons into the basic differentiation formulas, and maybe a few examples of integration by parts; the rest should be kicked up). They need not be facile at doing any complicated integrations or differentiations by hand (kicked out).

2) These students need not know about convergence of series except the minimum amount on the ratio test needed for Taylor series (rest kicked up, or, if the first discrete math course includes generating functions, placed there in part).

3) These students need not know about partial fraction decomposition of rational functions (kicked up, or kicked into the generating functions part of discrete math).

4) They need not know about parametric representations (kicked up) or polar and spherical coordinates (the former kicked up, the latter out).

5) They need not know quadratic analytic geometry beyond circles and vertical parabolas (rest kicked out).

6) They need not know trig formulas beyond sin and cos of a+b (kicked out).

7) In multivariate calculus they should know about partial derivatives and about the classical total derivative, but not about the derivative as a linear transformation (kicked up).

8) Theys need not know about abstract linear algebra, but only about constructive linear algebra in R (rest kicked up). This, of course, is not part of cutting back calculus; it is part of cutting back linear algebra so that it fits in the new one-year discrete course.

Such kicking really will result in a shorter calculus course. I know this because the books are already around: they go under names like "Short Calculus". The only mismatch is that these books have been aimed at weak students, not average and strong students.

Such kicking means the upper class years for the major must also be changed. Students will not get as far in analysis as the ones we sent to graduate mathematics departments in the last 20 years, but they will be much broader. It is a trade I am quite willing to make, even for the ones who still do go on to graduate departments.

In part I am willing to make the trade because I have high hopes for the increased understanding students will obtain in the first two years, if we do the new curriculum right. In calculus, we must see to it that they have a good feel for the behavior of standard functions, and for interpretations of derivatives, differences, integrals and sums. In linear algebra, they should know a lot about setting up and solving linear equations. More generally, they should have a good grasp of mathematical language. They should also have a good grasp of algorithmic language and ideas, and know something about intelligent use of computers. They should have experience with a wide variety of optimization problems and techniques, some of them heuristic. Finally, they should have experience with discrete topics like graphs and logic, and discrete/continuous topics like probability and statistics. If some understanding of all this is gained, to the point of having a reasonable grasp of how to apply it, I think other departments too will feel the change in curriculum is a worthy trade.

The idea promoted above — substantial excisions from calculus, not just streamlining, are necessary to make a one-year calculus course, and some of the excisions should reappear in the upperclass years — <u>is</u> in the Williamstown papers. It's just not put in flashing lights. I think one lesson is that it needs to be.

Suppose radical change in calculus doesn't happen. Might there at least be more gradual change? For instance, might one incorporate in the calculus sequence the discrete approach to limits, the simultaneous treatment of difference and differential equations, and more numerical analysis algorithms?

Yes, I think such changes will happen, whether or not more radical changes also catch on. Actually, over the years a number of calculus books with such ideas have been published. They haven't succeeded. But maybe the time wasn't right. Computing wasn't familiar and convenient enough, to the average faculty member as well as to students. Now it probably is. Incidentally, a recent effort along these lines, with reported good success, is the calculus course developed by Sheldon Gordon of Suffolk Community College, NY. He has written a text, but it's only available in note form. (Prof Gordon will attend this conference.)

Relevance to Two-Year Colleges

I have no expertise on this issue. Only recently have I come to understand (thanks to written and oral tutoring from Don Albers) that two-year and junior colleges vary considerably as to their educational objectives. I'm looking forward to gaining further understanding at this conference. In any event, I'm in no position to do more than make some general remarks on the relevance of the discrete/continuous debate to junior colleges -- remarks which may be obvious to you anyway.

Clearly, the relevance is different depending on whether a student is in a program aimed at transfer to a four-year institution or not. In the former case, you are pretty well constrained; you can't change your curriculum ahead of the four-year colleges and universities. Interestingly, in at least one state the constraint also goes the other way. Bettye Anne Case will report on how, in Florida, a university cannot require for admission to a department a course which is not offered at sufficiently many of the state two-year colleges which feed into that university. This means that either the two types of institutions work together on change, or it can't happen. (Bettye Anne is trying to ensure the former, which is certainly more fun for junior college faculty than waiting to follow the curriculum decisions of others.)

But what about nontransfer programs? Those of you involved in such programs have a wonderful opportunity. There may be even more incentive for you to increase discrete math than for the four-year institutions, and fewer roadblocks. I suspect that few of your students plan careers in engineering or hard science. Many of them may not need much calculus, certainly not much of the harder or deeper topics which would get left out in a one-year calculus course. So you can concentrate on revising the calculus and putting more discrete math in.

In his paper for Williamstown [8], Albers suggests that discrete math may have an additional value for remedial students, a especially significant group at many two-year colleges. Don argues that discrete math gives faculty something new to teach these students, accessible to them and interesting to learn, instead of teaching them the same things over again, only louder.

It seems to me we have to distinguish two extremes of students who place into remedial courses. First, there is the student who didn't learn the high school material for reasons independent of capability: the school was lousy, the home situation was chaotic, the student wasn't interested in learning then, the student had unfounded anxiety, etc. Second, there is the student whose ability is low: no matter how many times signed numbers are explained and practiced, no matter how much is done to overcome anxiety, he/she just can't

keep the mechanics straight.

The first sort of student, assuming the motivation is now there, probably would prefer to see the old stuff again, and master it. Adding new things as well, which better illustrate the usefulness of math, is fine so long as it is clear to the student that the old stuff is there too. The second sort of student, however, is not going to gain from seeing the old stuff again. Of course, the problem is how to tell the two types apart at the time of course registration.

For the second type, I'm not sure discrete vs continuous is the right issue. The issue should be: is there some way to teach these students concepts while leapfrogging the computations which they cannot do? The obvious answer is computers — symbolic computation, graphics, etc. It's certainly worth a try; not much else has worked.

But one must not be blindly optimistic. I pass along the observation my friend Joe Malkevitch (York College, CUNY) made when I suggested symbolic computation would save the day. Any software which does a subtle process must itself be somewhat subtle to use. For instance, differentiation being subtle, it's not surprising that (Df)g is different from D(fg), in which case the software must treat them differently and thus the person using the software must understand the difference. A person for whom the process is too hard may well find the task of correctly feeding the software no easier. If so, a software-aided course will get no closer to teaching concepts than the old course — it will simply bog down in pushing buttons instead of pushing pencils.

Let's hope this pessimistic vision is wrong. In any event, there is an opportunity for two-year colleges to be in the forefront of finding out.

REFERENCES

1. John G Kemeny, J Laurie Snell and Gerald L Thompson, "Introduction to Finite Mathematics," 3rd ed, Prentice-Hall, Englewood Cliffs NJ, 1974.

2. John G Kemeny, Thomas E Kurtz and J Laurie Snell, "Computing for a First Course in Finite Mathematics" and "Computing for a Second Course in Finite Mathematics," mimeographed, Dartmouth College, Hanover NH, 1983,

3. "The Future of College Mathematics: Proceedings of a Conference/Workshop on the First Two Years of College Mathematics," Anthony Ralston and Gail S Young, eds., Springer-Verlag, New York, 1983.

4. Tom M Apostol, "Calculus," Vol 1, 2nd ed, 1967 and Vol 2, 2nd ed, 1969, Wiley, New York. (Original editions published by Blaisdell, 1961 and 1962.)

5. Richard Johnsonbaugh, "Discrete Mathematics," Macmillan, New York, 1984.

6. Herbert S Wilf, Symbolic Manipulation and Algorithms in the Curriculum of the First Two Years, in [3], pp 27-42.

7. Donald E Knuth, "The Art of Computer Programming," Vol 1, 2nd Ed, Addison Wesley, 1973, pp 58.

8. Donald J Albers, The Impact of a New Curriculum on Remedial Mathematics, in [3], pp 191-199.

ERIC® CLEARINGHOUSE FOR
JUNIOR COLLEGES
UNIVERSITY OF CALIFORNIA

AUG 31 1984

8118 Math-Sciences Building
Los Angeles, California 90024